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Non-Fourier effects on transient temperature response in semitransparent medium caused by laser pulse

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Abstract

The non-Fourier effects on transient temperature response in semitransparent medium with black boundary surfaces caused by laser pulse are studied. The processes of the coupled conduction and radiation heat transfer in a one-dimensional semitransparent slab with black boundaries are analyzed numerically. The hyperbolic heat conduction equation is solved by the flux-splitting method, and the radiative transfer equation is solved by the discrete ordinate method. The transient temperature response obtained from hyperbolic heat conduction equation is compared with those obtained from the classical parabolic heat conduction equation. The results show that the non-Fourier effect can be important when the conduction-to-radiation parameter and the thermal relaxation time of heat conduction are larger. Under this condition, for the laser-flash measurement of the thermal diffusivity in semitransparent materials, omitting the non-Fourier effect can result in significant errors.

Keywords: Non-Fourier effect; Radiation–conduction coupled heat transfer; Transient heat transfer; Semitransparent medium

1. Introduction

Transient temperature response in solids subjected to a laser pulse has numerous practical applications, including the laser-flash measurement techniques of the thermal diffusivity in solids [1–4] and the use of high-energy pulse lasers [5–8]. In the analysis of heat conduction involving extremely short times, the classical heat conduction equation breaks down. Based on a relaxation model for heat conduction in solids and liquids, the traditional heat diffusion equation is replaced with a hyperbolic equation that accounts for the finite thermal propagation speed. The use of the hyperbolic equation removes the non-physical infinite speed of propagation predicted by the classical parabolic heat conduction equation [9]. Non-Fourier heat conduction in solids with different shapes and boundary conditions has been studied extensively [10–21]. Yuen and Lee [10], Tang and Araki [11] analyzed the non-Fourier heat conduction in a solid subjected to periodic thermal disturbances. Glass et al. [12–14] studied the non-Fourier effects on the transient temperature resulting from pulse heat flux in one-dimensional semi-infinite solid with linear or non-linear boundary condition. Ozisik and Tzou [15] analyzed the special features in thermal wave propagation, and the thermal wave model in relation to the microscopic two-step model. Tzou [16] presented a universal constitutive equation between the heat flux vector and the temperature gradient. Antaki [17] studied the non-Fourier effects in a semi-infinite slab with surface pulse heat flux by dual phase lag model. Lor and Chu [18] analyzed the effect of interface thermal resistance on heat transfer in a composite medium using the thermal wave model. Guo and Xu [19] studied the non-Fourier heat conduction in IC chip. Zhang and Liu [20] investigated the non-Fourier effects in spherical medium resulting from rapid temperature disturbance. Cai et al. [21] presented a one-dimensional explicit solution of non-Fourier heat conduction in spherical medium. All

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Nomenclature

C_p	specific heat, J/kg K
f	time-dependent laser intensity, W/m^2
F	non-dimensional time-dependent laser intensity, $F = f/4\bar{n}^2\sigma T_r^4$
I	intensity of radiation, W/m^2 sr
I^*	non-dimensional intensity of radiation, $I^* = \pi I/\bar{n}^2\sigma T_r^4$
k	thermal conductivity, W/m^2 K
L	slab thickness, m
$\bar{n}$	refractive index of medium
N	conduction-to-radiation parameter, $N = k\beta/4\bar{n}^2\sigma T_r^3$
q^c	heat flux of conduction, W/m^2
q^r	heat flux of radiation, W/m^2
q_p	laser pulse intensity, W/m^2
Q	non-dimensional heat flux, $Q = (q^c + q^r)/4\bar{n}^2\sigma T_r^4$
Q^c	non-dimensional heat flux of conduction, $Q^c = q^c/4\bar{n}^2\sigma T_r^4$
Q^r	non-dimensional heat flux of radiation, $Q^r = q^r/4\bar{n}^2\sigma T_r^4$
Q_p	non-dimensional laser pulse intensity, $Q_p = q_p/4\bar{n}^2\sigma T_r^4$
t	time, s
t_p	laser pulse time width, s
t_{tr}	thermal relaxation time of conduction, s
T	temperature of medium, K
T_r	temperature of surrounding, K
x	space coordinate, m

Greek symbols

α	thermal diffusivity, m^2/s
β	extinction coefficient, m^{-1}
θ	non-dimensional temperature, $\theta = T/T_r$
μ	direction cosine
ξ	non-dimensional time, $\xi = \alpha\beta^2 t$
ξ_p	non-dimensional laser pulse peak time, $\xi = \alpha\beta^2 t_p$
ξ_{tr}	non-dimensional thermal relaxation time, $\xi = \alpha\beta^2 t_{tr}$
ρ	density, kg/m^3
σ	Stefan–Boltzmann constant, W/m^2 K^4
τ	optical coordinate, $\tau = \beta x$
τ_L	optical thickness of slab, $\tau_L = \beta L$
Φ	scattering phase function
ω	single-scattering albedo

of these works focused on the non-Fourier effects in opaque solid medium.

Recently, the transient temperature response in semitransparent medium caused by laser pulse evoked the wide interests of many researchers. Tan et al. [22,23] investigated the temperature response caused by a pulse or a step laser irradiation in semitransparent slabs with generalized boundary conditions. They performed ray tracing using a band model and found that the laser-flash method in thermal metrology could give irrelevant results if radiation–conduction coupling was not properly taken into account in the heat transfer model. Andre and Degiovanni [2] studied the transient conduction–radiation heat transfer of non-scattering glass specimen. Hahn et al. [3] applied the three-flux method to calculate the temperature response caused by laser irradiation in an absorbing, isotropic scattering slab. Mehling et al. [4] developed a new analytical model to analyze the transient coupled conduction and radiation heat transfer processes in optically thin non-scattering semitransparent slabs, and applied the analytical model to determine the thermal diffusivity of semitransparent materials by the laser-flash method. All of these works did not consider the influence of non-Fourier heat conduction. To our knowledge, no work has investigated the influence of non-Fourier effects on coupled conduction and radiation heat transfer processes in semitransparent materials.

The objective of this work is to analyze the non-Fourier effect on transient temperature response in semitransparent medium caused by laser pulse. The coupled conduction and radiation heat transfer in a one-dimensional semitransparent slab with black boundaries is studied by numerical simulation: the hyperbolic heat conduction equation being solved by the flux-splitting method, and the radiative transfer equation being solved by the discrete ordinate method. For the sake of analysis, the transient temperature response obtained from hyperbolic heat conduction equation is compared with those obtained from the classical parabolic heat conduction equation. The influence of relating parameters, such as the pulse time width and the pulse intensity, on the transient temperature response will be analyzed.

2. Physical model and formulation

As illustrated in Fig. 1, we consider an absorbing, emitting, scattering, gray, one-dimensional semitransparent slab with black boundary surfaces, initially at thermal equilibrium with the surrounding. At time $t = 0$, a laser pulse is irradiated at the surface of $x = 0$. The

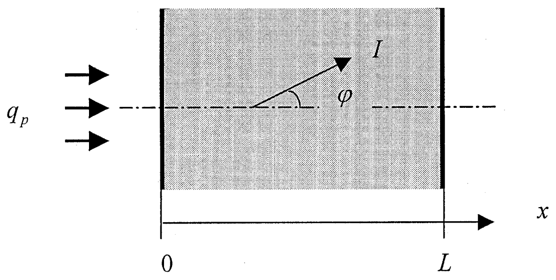

Fig. 1. Schematic of the physical model and coordinates.

energy equation for transient coupled radiative and conductive heat transfer in one-dimensional radiative participating medium is given by

$$\rho C_{\mathrm{p}} \frac{\partial T(x,t)}{\partial t} = -\frac{\partial q^{\mathrm{c}}(x,t)}{\partial x} - \frac{\partial q^{\mathrm{r}}(x,t)}{\partial x}, \tag{1a}$$

where ρ is the density of medium, C_{p} is the specific heat, T is the temperature, q^{c} is the conductive heat flux, q^{r} is the radiative heat flux, and x and t are the space and time coordinates, respectively. The modified Fourier equation [9], as formulated by Vernotte and Cattaneo, is

$$t_{\mathrm{tr}} \frac{\partial q^{\mathrm{c}}(x,t)}{\partial t} + q^{\mathrm{c}}(x,t) = -k \frac{\partial T(x,t)}{\partial x}, \tag{1b}$$

where k is the thermal conductivity and t_{tr} is the thermal relaxation time. The boundary and initial conditions are given as:

$$q^{\mathrm{c}}(0,t) + q^{\mathrm{r}}(0,t) = f(t) - \sigma\left[T^4(0,t) - T_{\mathrm{r}}^4\right], \quad x = 0, \tag{1c}$$

$$q^{\mathrm{c}}(L,t) + q^{\mathrm{r}}(L,t) = \sigma\left[T^4(L,t) - T_{\mathrm{r}}^4\right], \quad x = L, \tag{1d}$$

$$q^{\mathrm{c}}(x,0) = 0, \quad t = 0, \tag{1e}$$

$$T(x,0) = T_{\mathrm{r}}, \quad t = 0, \tag{1f}$$

where T_{r} is the temperature of surrounding, $f(t)$ is the time-dependent laser pulse intensity.

In this paper, we consider the laser pulse represented mathematically in the form

$$f(t) = \begin{cases} q_{\mathrm{p}}, & 0 \leqslant t \leqslant t_{\mathrm{p}}, \\ 0, & t > t_{\mathrm{p}}, \end{cases} \tag{2}$$

where q_{p} is the laser pulse intensity, and t_{p} is the pulse time width.

For semitransparent media, such as glass, the relaxation time of radiation is less than that of conduction by many orders of magnitude, so the wave feature of radiation can be omitted. The equation of radiative transfer in an absorbing, emitting, scattering, gray, one-dimensional semitransparent slab with black boundaries can be written as [24]

$$\mu \frac{\partial I(x,\mu,t)}{\partial x} = -\beta I(x,\mu,t) + \frac{\beta\omega}{2} \int_{-1}^{1} I(x,\mu',t) \Phi(\mu,\mu')\, \mathrm{d}\mu' + (1-\omega)\beta I_{\mathrm{b}}(x,t) \tag{3a}$$

with boundary conditions

$$I(0,\mu,t) = \frac{\bar{n}^2 \sigma T^4(0,t)}{\pi}, \quad 0 \leqslant \mu \leqslant 1, \ x = 0, \tag{3b}$$

$$I(L,\mu,t) = \frac{\bar{n}^2 \sigma T^4(L,t)}{\pi}, \quad -1\mu \leqslant 0, \ x = L, \tag{3c}$$

where I is the radiation intensity, σ is the Stefan–Boltzmann constant, ω is the single-scattering albedo, β is the extinction coefficient, $\bar{n}$ is the refractive index of medium, $\mu = \cos\varphi$ is the direction cosine, and Φ is the scattering phase function. The radiative heat flux can be expressed as

$$q^{\mathrm{r}}(x,t) = 2\pi \int_{-1}^{1} I(x,\mu,t)\mu\, \mathrm{d}\mu. \tag{4}$$

Eqs. (1a)–(1f), (2),(3a)–(3c), (4) are non-dimensionalized, and then, we have:

$$\frac{\partial \theta(\tau,\xi)}{\partial \xi} + \frac{1}{N} \frac{\partial Q(\tau,\xi)}{\partial \tau} = 0, \tag{5a}$$

$$\frac{\partial Q(\tau,\xi)}{\partial \xi} + \frac{N}{\xi_{\mathrm{tr}}} \frac{\partial \theta(\tau,\xi)}{\partial \tau} + \frac{1}{\xi_{\mathrm{tr}}} Q(\tau,\xi) - \frac{\partial Q^{\mathrm{r}}(\tau,\xi)}{\partial \xi} - \frac{1}{\xi_{\mathrm{tr}}} Q^{\mathrm{r}}(\tau,\xi) = 0, \tag{5b}$$

$$Q(0,\xi) = F(\xi) - \frac{1}{4\bar{n}^2}\left[\theta^4(0,\xi) - 1\right], \quad \tau = 0, \tag{5c}$$

$$Q(\tau_L,\xi) = \frac{1}{4\bar{n}^2}\left[\theta^4(\tau_L,\xi) - 1\right], \quad \tau = \tau_L, \tag{5d}$$

$$Q(\tau,0) = 0, \quad \xi = 0, \tag{5e}$$

$$\theta(\tau,\xi) = 1, \quad \xi = 0, \tag{5f}$$

$$F(\xi) = \begin{cases} Q_{\mathrm{p}}, & 0 \leqslant \xi \leqslant \xi_{\mathrm{p}} \\ 0, & \xi > \xi_{\mathrm{p}}, \end{cases} \tag{6}$$

$$\mu \frac{\partial I^*(\tau,\mu,\xi)}{\partial \tau} = -I^*(\tau,\mu,\xi) + \frac{\omega}{2} \int_{-1}^{1} I^*(\tau,\mu',\xi) \Phi(\mu,\mu')\, \mathrm{d}\mu' + (1-\omega)\theta^4(\tau,\xi), \tag{7a}$$

$$I^*(0,\mu,\xi) = \theta^4(0,\xi), \quad 0 \leqslant \mu \leqslant 1, \ \tau = 0, \tag{7b}$$

$$I^*(\tau_L,\mu,\xi)=\theta^4(\tau_L,\xi),\quad -1\leqslant\mu\leqslant 0,\ \ \tau=\tau_L, \tag{7c}$$

$$Q^{\mathrm{r}}=\frac{1}{2}\int_{-1}^{1}I^*(\tau,\mu,\xi)\mu\,\mathrm{d}\mu \tag{8}$$

where

$$N=\frac{k\beta}{4\bar{n}^2\sigma T_{\mathrm{r}}^3}, \tag{9a}$$

$$Q(\tau,\xi)=\frac{q^{\mathrm{c}}+q^{\mathrm{r}}}{4\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9b}$$

$$Q^{\mathrm{r}}(\tau,\xi)=\frac{q^{\mathrm{r}}}{4\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9c}$$

$$Q^{\mathrm{c}}(\tau,\xi)=\frac{q^{\mathrm{c}}}{4\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9d}$$

$$Q_{\mathrm{p}}=\frac{q_{\mathrm{p}}}{4\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9e}$$

$$F(\xi)=\frac{f}{4\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9f}$$

$$I^*=\frac{\pi I}{\bar{n}^2\sigma T_{\mathrm{r}}^4}, \tag{9g}$$

$$\theta(\tau,\xi)=\frac{T}{T_{\mathrm{r}}}, \tag{9h}$$

$$\xi=\alpha\beta^2 t, \tag{9i}$$

$$\xi_{\mathrm{p}}=\alpha\beta^2 t_{\mathrm{p}}, \tag{9j}$$

$$\xi_{\mathrm{tr}}=\alpha\beta^2 t_{\mathrm{tr}}, \tag{9k}$$

$$\tau=\beta x, \tag{9l}$$

$$\tau_L=\beta L. \tag{9m}$$

3. Method for solution

The energy equation and non-Fourier constitutive equation is solved numerically by flux-splitting method [25]. To use the flux-splitting method, Eqs. (5a) and (5b) are first written in vector form as

$$\frac{\partial\mathbf{U}}{\partial\xi}+\frac{\partial\mathbf{E}}{\partial\tau}=\mathbf{S}, \tag{10}$$

where

$$\mathbf{U}=\begin{bmatrix}\theta\\ Q\end{bmatrix},\quad \mathbf{E}=\begin{bmatrix}\frac{Q}{N}\\ \frac{N\theta}{\xi_{\mathrm{tr}}}\end{bmatrix},\quad \mathbf{S}=\begin{bmatrix}0\\ \frac{Q^{\mathrm{r}}}{\xi_{\mathrm{tr}}}+\frac{\partial Q^{\mathrm{r}}}{\partial\xi}-\frac{Q}{\xi_{\mathrm{tr}}}\end{bmatrix}. \tag{11}$$

In splitting the flux terms **E**, the flux is assumed to be composed of a positive and a negative component, each associated with the signal propagation directions. The vector equation can also be rewritten in the form

$$\frac{\partial\mathbf{U}}{\partial\xi}+[A]\frac{\partial\mathbf{U}}{\partial\tau}=\mathbf{S}, \tag{12}$$

where $[A]$ is the Jacobian $\partial\mathbf{E}/\partial\mathbf{U}$ given by

$$[A]=\begin{bmatrix}0 & \frac{1}{N}\\ \frac{N}{\xi_{\mathrm{tr}}} & 0\end{bmatrix}. \tag{13}$$

Because the equation is hyperbolic, a similarity transformation exists so that

$$[A]=[T][\lambda][T]^{-1}, \tag{14}$$

where $[\lambda]$ is a diagonal matrix of real eigenvalues of $[A]$, $[T]^{-1}$ is the matrix whose rows are the left eigenvectors of $[A]$ taken in order. The vector can be expressed as

$$\mathbf{E}=[A]\mathbf{U}=[T][\lambda][T]^{-1}\mathbf{U}, \tag{15}$$

where

$$[T]=\begin{bmatrix}1 & 1\\ -\frac{N}{\sqrt{\xi_{\mathrm{tr}}}} & \frac{N}{\sqrt{\xi_{\mathrm{tr}}}}\end{bmatrix}, \tag{16}$$

$$[T]^{-1}=\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{\xi_{\mathrm{tr}}}}{2N}\\ \frac{1}{2} & \frac{\sqrt{\xi_{\mathrm{tr}}}}{2N}\end{bmatrix}, \tag{17}$$

$$[\lambda]=\begin{bmatrix}-\frac{1}{\sqrt{\xi_{\mathrm{tr}}}} & 0\\ 0 & \frac{1}{\sqrt{\xi_{\mathrm{tr}}}}\end{bmatrix}. \tag{18}$$

The matrix of eigenvalues is divided into two matrices, one with only positive elements and the other with negative elements. We can write $[A]$ matrix as

$$[A]=[A^+]+[A^-]=[T][\lambda^+][T]^{-1}+[T][\lambda^-][T]^{-1} \tag{19}$$

and define

$$\mathbf{E}=\mathbf{E}^++\mathbf{E}^- \tag{20}$$

so that

$$\mathbf{E}^+=[A^+]\mathbf{U},\quad \mathbf{E}^-=[A^-]\mathbf{U}, \tag{21}$$

where

$$[A^+]=\begin{bmatrix}\frac{1}{2\sqrt{\xi_{\mathrm{tr}}}} & \frac{1}{2N}\\ \frac{N}{2\xi_{\mathrm{tr}}} & \frac{1}{2\sqrt{\xi_{\mathrm{tr}}}}\end{bmatrix}, \tag{22}$$

$$[A^-] = \begin{bmatrix} -\frac{1}{2\sqrt{\xi_{tr}}} & \frac{1}{2N} \\ \frac{N}{2\xi_{tr}} & -\frac{1}{2\sqrt{\xi_{tr}}} \end{bmatrix}. \tag{23}$$

The vector equation (10) can be written as the split-flux notation

$$\frac{\partial \mathbf{U}}{\partial \xi} + \frac{\partial \mathbf{E}^+}{\partial \tau} + \frac{\partial \mathbf{E}^-}{\partial \tau} = \mathbf{S}, \tag{24}$$

where the plus and minus signs indicate that the flux components are associated with wave propagation in the positive and negative directions, respectively. The following finite-difference formulation can be obtained by applying backward- and forward-difference schemes to $\mathbf{E}^+$ and $\mathbf{E}^-$, respectively:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta\xi}{\Delta\tau}\left[(\mathbf{E}^+)_i^n - (\mathbf{E}^+)_{i-1}^n\right] - \frac{\Delta\xi}{\Delta\tau}\left[(\mathbf{E}^-)_{i+1}^n - (\mathbf{E}^-)_i^n\right] + \Delta\xi \mathbf{S}_i^n. \tag{25}$$

Substituting Eq. (21) into Eq. (25), we have

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta\xi}{\Delta\tau}[A^+]\left[\mathbf{U}_i^n - \mathbf{U}_{i-1}^n\right] - \frac{\Delta\xi}{\Delta\tau}[A^-]\left[\mathbf{U}_{i+1}^n - \mathbf{U}_i^n\right] + \Delta\xi \mathbf{S}_i^n. \tag{26}$$

Here, subscript i denotes the grid point in space domain, superscript n denotes the time level, and $\Delta\tau$ and $\Delta\xi$ are the space and time steps, respectively.

Eqs. (7a)–(7c) for radiative transfer are solved by the discrete ordinate method and S_8 results are obtained. In this approach, the solid angle is discretized into a finite number of directions. The equation of radiative transfer is evaluated at each of the discrete directions, and the integral of in-scattering term is replaced by a weighted sum, which leads to the discrete ordinate equations. The solution to the discrete ordinate equations of radiative transfer equation can be obtained iteratively. The detailed solution procedure can be seen in Ref. [26] and will not be repeated here.

For the purposes of comparison, the dimensionless form of the parabolic heat conduction equation in an absorbing, emitting, scattering, gray, one-dimensional semitransparent slab with black boundaries is written as

$$\frac{\partial\theta(\tau,\xi)}{\partial\xi} = \frac{\partial^2\theta(\tau,\xi)}{\partial\tau^2} - \frac{1}{N}\frac{\partial Q^r(\tau,\xi)}{\partial\tau} \tag{27a}$$

with boundary and initial conditions:

$$\frac{\partial\theta(0,\xi)}{\partial\tau} = \frac{1}{N}\left[Q^r(0,\xi) - F(\xi) + \frac{1}{4\bar{n}^2}\left[\theta^4(0,\xi) - 1\right]\right], \quad \tau = 0, \tag{27b}$$

$$\frac{\partial\theta(\tau_L,\xi)}{\partial\tau} = \frac{1}{N}\left[Q^r(\tau_L,\xi) - \frac{1}{4\bar{n}^2}\left[\theta^4(\tau_L,\xi) - 1\right]\right], \quad \tau = \tau_L, \tag{27c}$$

$$\theta(\tau,0) = 1, \quad \xi = 0. \tag{27d}$$

An implicit central-difference scheme [27] is used to solve the parabolic heat conduction equation.

4. Results and discussions

A computer code was written based on the above calculation procedure. Grid refinement and time step sensitivity studies were also performed for the physical model to ensure that the essential physics are independent of grid size and time interval. The scattering phase function, $\Phi(\mu,\mu') = 1 + 0.5\mu\mu'$, was used. Due to the nonlinear nature of the model equation and its boundary conduction, the Courant number less than 0.01 was used in solving hyperbolic heat conduction equation for all case in this paper.

For homogeneous materials, such as pure liquids, gases and dielectric solids, the values of thermal relaxation time range from 10^{-14} to 10^{-6} s [28–30]. However, the values of thermal relaxation time might be significantly larger for materials with a non-homogeneous inner structure. The extinction coefficient, β, for glass can range from 20 to 2000 m^{-1}. Based on the thermal diffusivity and thermal relaxation time data of Kaminski [28], the non-dimensional thermal relaxation time, ξ_{tr}, can range from 0.001 to 10.

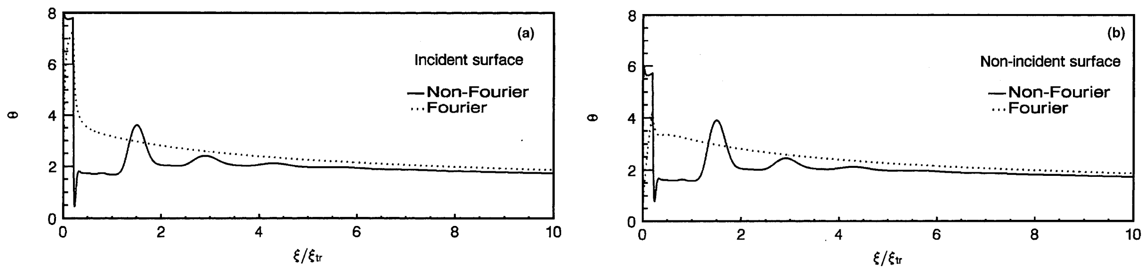

Fig. 2. Transient temperature response of the incident and the non-incident surfaces for the case of $N = 5$, $n = 1$, $\tau_L = 0.1$, $\omega = 0.5$, $Q_p = 2000$, $\xi_{tr} = 0.005$ and $\xi_p = 0.2\xi_{tr}$.

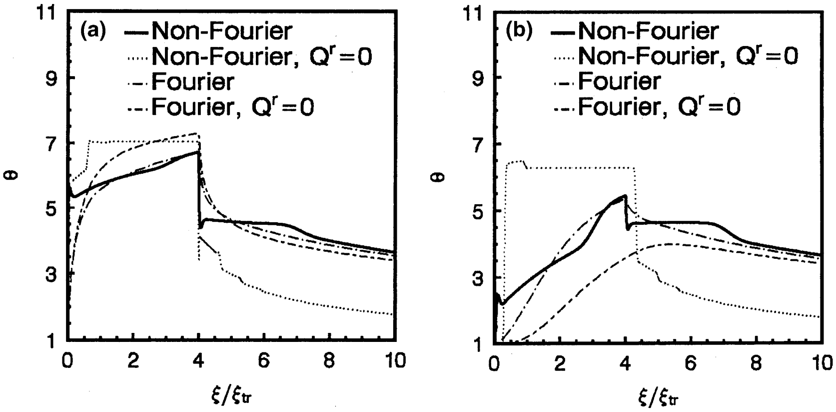

Fig. 3. The influence of media radiation on the transient temperature responses: (a) incident surface; (b) non-incident surface.

Fig. 2 shows the influence of non-Fourier effect on the transient temperature response of the incident and the non-incident surfaces for the case of $N = 5$, $n = 1$, $\tau_L = 0.1$, $\omega = 0.5$, $Q_p = 2000$, $\xi_{tr} = 0.005$ and $\xi_p = 0.2\xi_{tr}$. The temperature responses, obtained from non-Fourier heat conduction equations for both the incident and non-incident surfaces, oscillate. At the time of $\xi/\xi_{tr} = 0.2$, numerical false diffusion appeared near the discontinuities of laser pulse. This is the numerical feature of flux-splitting method at the discontinuities and does not affect the solution at the other time. In the initial stage, the differences between the solutions ob-

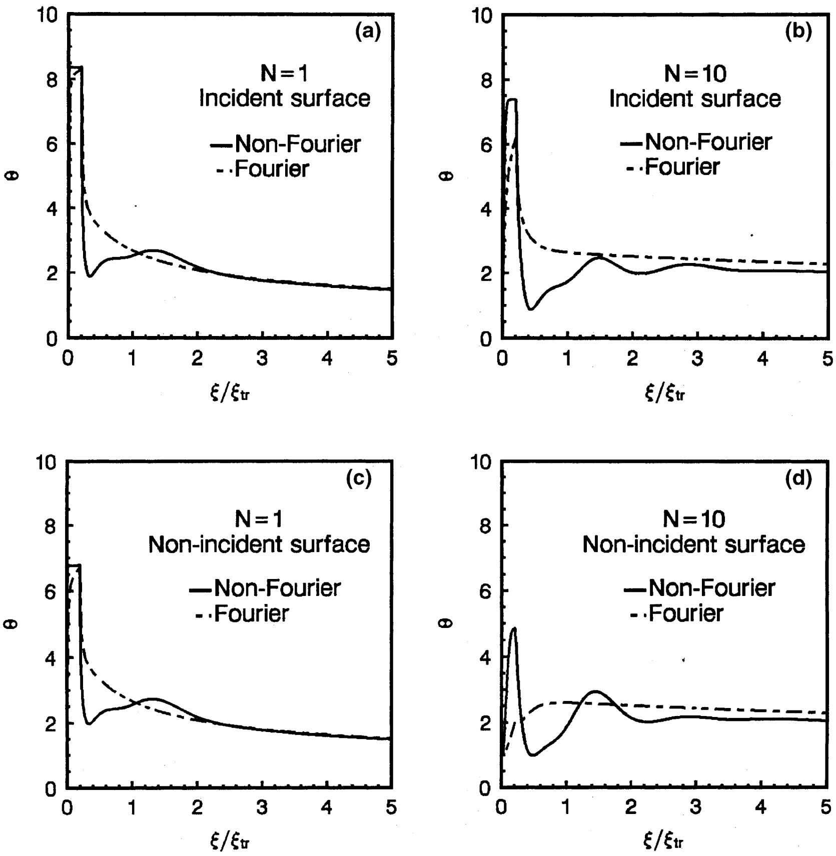

Fig. 4. The influences of conduction-to-radiation parameter on the non-Fourier effects of transient temperature responses.

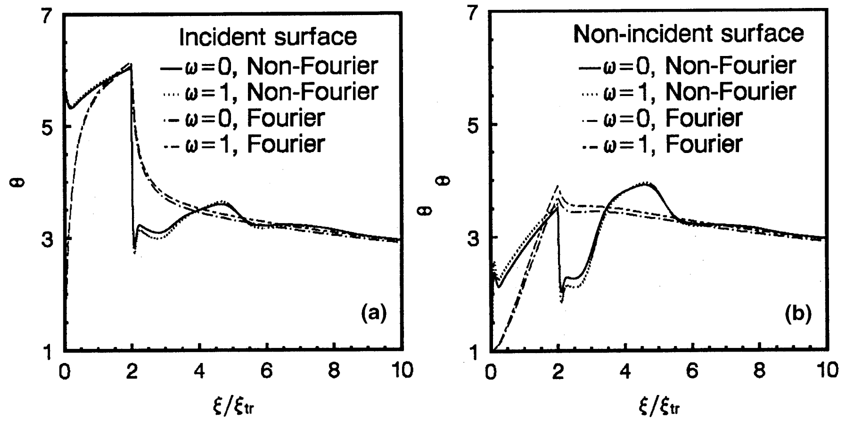

Fig. 5. The influences of single-scattering albedo on the non-Fourier effects of transient temperature responses.

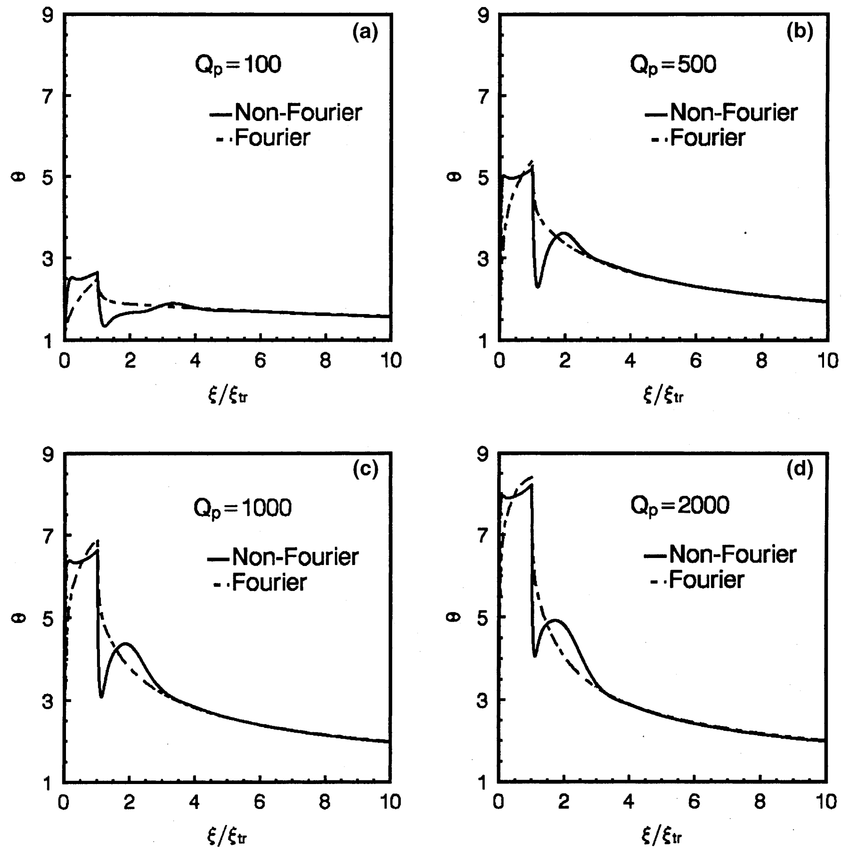

Fig. 6. The influences of laser pulse intensity on non-Fourier effects of transient temperature responses of the incident surface.

tained from hyperbolic equations and those obtained from parabolic equations are larger, and the non-Fourier effect cannot be omitted. After a certain time, the non-Fourier effect diminishes, and the solutions obtained from hyperbolic equations converge to those obtained from parabolic equations. In general, the non-Fourier effect is shown to decay quickly, and the conventional Fourier equation is accurate a short time after the initial transient.

Fig. 3 shows the influence of media radiation on the transient temperature response of the incident and the non-incident surfaces for the case of $N = 5$, $n = 1$,

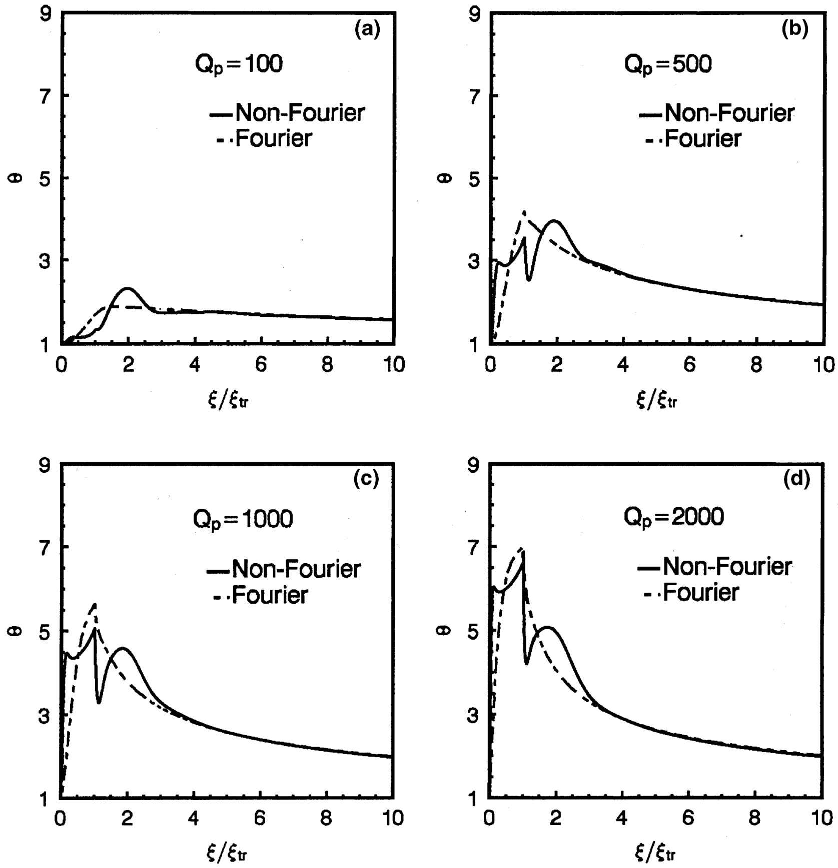

Fig. 7. The influences of laser pulse intensity on non-Fourier effects of transient temperature responses of non-incident surface.

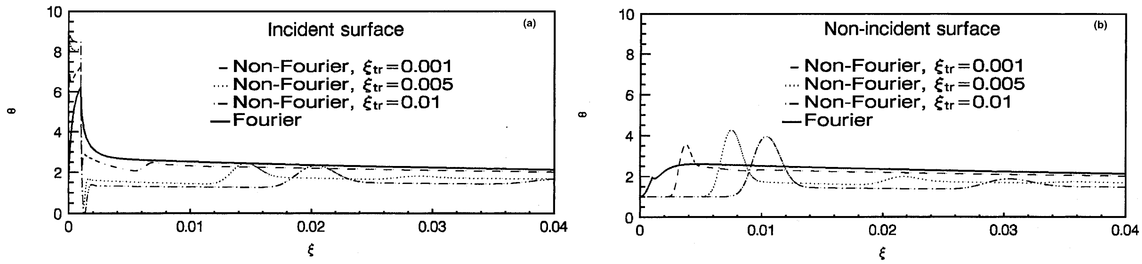

Fig. 8. The influences of thermal relaxation time of heat conduction on the non-Fourier effects of transient temperature responses.

$\tau_L = 0.1$, $\omega = 0.5$, $Q_p = 1000$, $\xi_{tr} = 0.001$ and $\xi_p = 4\xi_{tr}$. The results obtained from parabolic equations for the non-radiation case (i.e. $Q^r = 0$) are also included. The conduction-to-radiation parameter N characterizes the relative importance of conduction in regard to radiation. As shown in Fig. 3, for the case of $N = 5$, heat conduction is dominated by comparison with heat radiation, the differences between the calculation results of temperature responses for the radiation and the non-radiation cases are significant, and media radiation reduces enormously the non-Fourier effects in semitransparent medium. Even so, for semitransparent medium, the application of the laser-flash method in thermal metrology may give irrelevant results if the basic heat transfer model is not able to take into account non-Fourier effects of heat conduction. Even though the time width of laser pulse was widened by comparison with Fig. 2, the non-Fourier effects are significant in the initial stage of laser pulse on-and-off processes.

Fig. 4 shows the influences of conduction-to-radiation parameter on the non-Fourier effects of transient temperature responses in the case of $\overline{n} = 1$, $\tau_L = 0.1$, $\omega = 0.5$, $Q_p = 2000$, $\xi_{tr} = 0.005$ and $\xi_p = 0.2\xi_{tr}$. Increasing conduction-to-radiation parameter N, the non-Fourier effects on the transient temperature responses of the incident and the non-incident surface increase obviously. This is due to the decreasing of media radiation with the increasing of conduction-to-radiation parameter.

The influences of single-scattering albedo on the non-Fourier effects of transient temperature responses in the case of $N = 5$, $\overline{n} = 1$, $\tau_L = 0.1$, $Q_p = 1000$, $\xi_{tr} = 0.001$ and $\xi_p = 2\xi_{tr}$ are shown in Fig. 5, from which the influences of single-scattering albedo are small.

The influences of laser pulse intensity on non-Fourier effects of transient temperature responses in the case of $N = 5$, $\overline{n} = 1$, $\tau_L = 0.1$, $\omega = 0.5$, $\xi_{tr} = 0.005$ and $\xi_p = \xi_{tr}$ are shown in Fig. 6 for incident surface and Fig. 7 for non-incident surface. The non-dimensional laser pulse intensity Q_p changes from 100 to 2000. Increasing the non-dimensional laser pulse intensity, the non-Fourier effects on the transient temperature responses of the incident and the non-incident surface increase in the initial stage of on-and-off processes of laser pulse.

Fig. 8 shows the influences of thermal relaxation time of heat conduction on the non-Fourier effects of transient temperature responses in the case of $N = 10$, $\overline{n} = 1$, $\tau_L = 0.1$, $\omega = 0.5$, $Q_p = 2000$ and $\xi_p = 0.001$. As shown in Fig. 8, the non-Fourier effects of transient temperature responses of the incident and the non-incident surfaces increase with the thermal relaxation time of heat conduction. When ξ_{tr} is increased from 0.001 to 0.01, the phase lags of thermal wave on the non-incident surface increases obviously.

5. Conclusions

The non-Fourier effects on transient temperature response in semitransparent medium with black boundary surfaces caused by laser pulse are studied. The processes of the coupled conduction and radiation heat transfer in a one-dimensional semitransparent slab with black boundaries are analyzed numerically. The hyperbolic heat conduction equation is solved by the flux-splitting method, and the radiative transfer equation is solved by the discrete ordinate method. The transient temperature response obtained from hyperbolic heat conduction equation is compared with those obtained from the classical parabolic heat conduction equation. The main conclusions can be summarized as follows:

1. In the range of parameters studied in this paper, the temperature responses obtained from non-Fourier heat conduction equations for both the incident and non-incident surfaces oscillate. In the initial stage of laser pulse, the differences between and the solutions obtained from hyperbolic equations and those obtained from parabolic equations are larger, so that the non-Fourier effect cannot be omitted. In general, the non-Fourier effect is shown to decay quickly, and the conventional Fourier equation becomes accurate a short time after the initial transient.

2. The media radiation reduces enormously the non-Fourier effects in semitransparent medium. Even so, for semitransparent medium, the application of the laser-flash method in thermal metrology may give irrelevant results if the basic heat transfer model is not able to take into account non-Fourier effects of heat conduction. The non-Fourier effects are significant in the initial stage of laser pulse on-and-off processes.

3. Increasing non-dimensional laser pulse intensity, the non-Fourier effects on the transient temperature responses of the incident and the non-incident surface increase in the initial stage of on-and-off processes of laser pulse. The non-Fourier effect can be important when the conduction-to-radiation parameter and the thermal relaxation time of heat conduction are larger, and hence, for the laser-flash measurement of the thermal diffusivity in semitransparent materials, omitting the non-Fourier effect may result in significant errors.

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